AOS 610 Geophysical Fluid Dynamics I Prof. Hitchman
Problem Set #4 10 points Due Thursday December 5, 2024

1. (1.5 pts) *Orographic gravity waves.*
a) (1 pt) Determine the perturbation horizontal and vertical velocity fields

$u' = u_o \sin(kx + mz), \quad w' = w_o \sin(kx + mz)$ (solve for u_o, w_o, k, m).

b) (0.5 pt) Evaluate the magnitude and sign of the vertical flux of westerly momentum $\rho \overline{u'w'}$ (Pa) for stationary gravity waves forced by westerly flow over sinusoidally-varying topography $h = h_o \cos kx$, where h_o = 100 m, N = 2 x 10^{-2} s^{-1}, $\bar{u}$ = 5 m/s, and L_x = 5 km. We can approximate the lower boundary condition as $w'(0) = dh/dt = \bar{u}\frac{\partial u}{\partial x}$. You may wish to use the definite integral $\int_0^{2\pi} sin^2 x\, dx = \pi$.

2. (1.5 pts) *Two layer baroclinic mode in the ocean.*
The water surface is at height η above the bottom of the ocean and there is a thermocline at height h above the bottom of the ocean. Let the upper layer be warmer than the lower layer by 10 K and have a depth of 200 m, with a total depth of 4 km. You may wish to use Table A3.1 of Gill with salinity 35 ppt to estimate densities in the two layers.
a) Estimate the baroclinic equivalent depth, $H_e = \frac{\rho_2 - \rho_1}{\rho_2} H_1$, and corresponding phase speed $c_1 = \sqrt{\hat{g}\, H_1} = \sqrt{g\, H_e}$ (cf. Gill 6.2.22).
b) For the baroclinic mode, estimate the ratio $|\eta|/|h|$ using $\frac{h}{\eta} = 1 - \frac{g\, H_1}{c_e^2}$ (Gill 6.2.13), and compare it to the magnitude of $\frac{\rho_2 - \rho_1}{\rho_2}$.

3. (1 pt) *Rossby radius of deformation.*
a) Assuming that a typical gravity wave emanating from a developing midlatitude cyclone has a vertical wavelength of about twice the depth of the tropopause, estimate the baroclinic Rossby radius of deformation, $L_R = N/|mf|$.
b) Considering shallow water theory, $L_R = {}^{c}/_{f}$, where $c = \sqrt{\frac{\rho_2 - \rho_1}{\rho_2} g\, H_1}$, why is L_R ~10 times bigger in the atmosphere than in the ocean?

4. (2 pts) *Observed Rossby wave propagation and the dispersion relation.*
How accurate is the Rossby wave dispersion relation? Pick a wave feature of interest, track its eastward progress for a few days, and compare it with theory. Go to http://www.aos.wisc.edu/weather/Models to view NCEP GFS Northern Hemisphere polar stereographic projections of weather forecasts and look for troughs and ridges in the white geopotential height contours at 300 hPa. You might also check 500 hPa and 250 hPa.

a) Estimate the observed zonal trace speed, c_{obs}, of a chosen midlatitude trough or ridge from a sequence of synoptic charts. Attach sample charts and indicate your chosen trough and ridge axes. Estimate how fast they travelled along a latitude circle during your observation time, δt, using $c_{obs} \approx \delta x/\delta t$, where $\delta x = a \cos\phi \ \delta\lambda * \pi/180$, with $\delta\lambda$ in degrees longitude. You may need to consult a globe or map to find latitude and longitude.

b) Compare your observed estimate in b) with an estimate from the theoretical zonal trace speed, c_{theory}, obtained from the linear barotropic dispersion relation. You will need to estimate the zonal and meridional wavelengths (k and l), assuming an infinite vertical wavelength ($m = 0$). You will also need to estimate the average zonal wind, $\bar{u}$, throughout the region where your wave exists, and choose a central latitude for estimating β and $\cos\phi$ from an auxilliary map. Please attach a few screen shots to show the evolution of your troughs/ridges.

5. (2.5 pts) *Rossby-ish waves in a rotating tank.*
The angular frequency is Ω and the fluid depth decreases toward the center as $H(y) = H_o - \gamma\, y$, due to a sloping bottom with slope γ. [In this tank y increases toward the rotation axis and z increases downward, so v > 0 is toward the center, but w > 0 is downward.]

a) Show that the perturbation continuity equation can be written $H_o\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) - \gamma\, v' = 0$

b) and that the perturbation potential vorticity equation is $\frac{\partial}{\partial t}\nabla^2\psi' + \beta\frac{\partial \psi'}{\partial x} = 0$.
Thus, the sloping bottom gives an "equivalent beta effect".

c) What is the Rossby wave phase speed for waves of wavelength 50 cm in the azimuthal (local Cartesian x direction) if $\Omega = 1\ s^{-1}$, $H_o = 20$ cm, and $\gamma = 0.1$?

6. (1 pt) *Rossby wave energy propagation in the meridional plane*
a) Starting from $\omega = \bar{u}\, k - \frac{\beta\, k}{k^2+l^2+\frac{f^2}{N^2}m^2} = \bar{u}k - \frac{\beta\, k}{K^2}$, show that $\frac{G_y}{G_z} = \frac{N^2}{f^2}\frac{l}{m}$.
b) If Rossby wave phase surfaces (troughs and ridges) are observed to tilt westward with increasing altitude and eastward with increasing latitude, sketch the path of energy propagation in the latitude altitude plane for the Northern Hemisphere, labelling your axes.

7. (0.5 pt) *Inertial instability*
Starting from the definition of zonal mean angular momentum per unit mass, $\bar{m} = a\cos\varphi\,(\Omega a\cos\varphi + \bar{u})$, show that the criterion for inertial instability for the zonal mean zonal flow in the NH, $f - \frac{1}{\cos\phi}\frac{\partial(\cos\phi\ \bar{u})}{\partial y} < 0$ can be written as $-\frac{1}{\cos\phi}\frac{\partial\bar{m}}{\partial y} < 0$, hence that $\frac{\partial\bar{m}}{\partial y} > 0$ indicates inertial instability (angular momentum increases toward the rotation axis).