AOS 610 GFD I Fall 2024 Prof. Hitchman Name ____________________________

Problem Set #3 Due Tuesday, November 19, 2024

1. (1 pt) *Static stability*.

Starting from the definition of potential temperature, it can be shown that

$$N^2 \equiv \frac{g}{\theta}\frac{\partial\theta}{\partial z} = \frac{g}{T}\left(\frac{\partial T}{\partial z} + \Gamma_d\right)$$

where N is the buoyancy frequency and $\Gamma_d = g/c_p$ is the dry adiabatic lapse rate. Using the attached soundings at Singapore and Green Bay, WI, estimate the rate of change of potential temperature with height $\frac{\partial\theta}{\partial z}$ in K/km and the buoyancy period $\tau_B = 2\,\pi/N$ in minutes, where N is the buoyancy frequency (s^{-1}), for the following regions:

a) tropical upper troposphere (~ 400 - 100 hPa) in the Maritime Continent region, and

b) midlatitude lower troposphere (~1000 - 500 hPa) in the warm conveyor belt of the "Halloween snowstorm" of 2019.

2. (1 pt) *Conservation of absolute vorticity*.

A tropical air mass over the Atlantic near 30°N moves northward conserving absolute vorticity.

a) If its initial relative vorticity is zero, what will it be at 60°N?

b) If the air mass has a 500-km radius and is moving in solid body rotation, what are the tangential wind speed and the sense of rotation at 60°N?

3. (1.5 pts) *Ertel's potential vorticity*.

From Fig. 9.1a in Andrews, Holton and Leovy, estimate zonal mean Ertel's PV at

a) 45°N, 500 hPa and b) 45°N, 100 hPa. Express your answer in PV units, where 1 PVU = $1\text{x}10^{-6}$ K-m^2-kg^{-1}-s^{-1}. Check your answers against Fig. 9.1b. Note that values in Fig. 9.1b are in units of $1\text{x}10^{-5}$ K-m^2-kg^{-1}-s^{-1}, or 10 PVU.

c) How do the distributions of ozone and PV differ above 20 km? What causes this difference?

4. (2 pts) *Isentropic PV charts*.

a) Create polar stereographic maps of PV on the 350 K and 850 K isentropic surfaces for 0000 UTC 29 December 2020 using PANOPLY and JRA3Q isentropic charts from the following (780 Mb) file available under homework 3 on our website:
jra3q.anl_isentrop.0_2_14.pvort-theta-an-gauss.2020122100_2020123118.nc.

b) Describe geographically where the 350 K PV pattern indicates a blocking high. Explain why you think so.

c) Does the 850 K PV pattern satisfy the criterion for Rossby wave breaking? Explain why.

5. (2.5 pts) *Ocean surface layer energetics*. Consider a well-mixed oceanic boundary layer of depth 20 m near the equator. The wind stress is given by $\tau = \rho\, c_d u^2$, where c_d = 0.001, ρ is air density, and u is the wind speed.

a) Assuming that there is no friction between the well-mixed layer and the deep ocean, and that all of the energy goes into kinetic energy of the slab upper ocean, how long would it take for the ocean current v to reach the speed of the air, assuming that u = 5 m/s in the above formula? [Assume a slab of depth h. The momentum change in the slab with net viscous force $\frac{\partial \tau}{\partial z}$ gives $\rho_w$dv/dt = ($\rho\, c_d u^2$- 0)/h.]

b) Consider what happens in this formulation when the current speed reaches v = 5 m/s. How would you modify the relation $\tau = \rho\, c_d u^2$, to take into account the current speed v?

c) When the current speed reaches 1 m/s, what is the rate of generation of kinetic energy per unit mass in the slab? [Multiply your solution in a) by v to form an equation for time rate of change of KE in the upper slab of the ocean.]

d) In steady state one may assume that the energy input by the wind stress is balanced by the viscous dissipation rate per unit mass, η. Using your answer from c), estimate the length scale at which deformation work balances molecular diffusion.

[The Kolmogorov microscale is $l^* = \left(\frac{\nu^3}{\eta}\right)^{1/4}$; use ν for water.]

e) Taking a typical value of c_p for the ocean from Table A3.1 of Gill, what is the rate of change of temperature associated with this kinetic energy input by wind stress? [Use conservation of energy in the presence of viscous dissipation $c_p \frac{\mathrm{dT}}{\mathrm{dt}} = \eta$.]

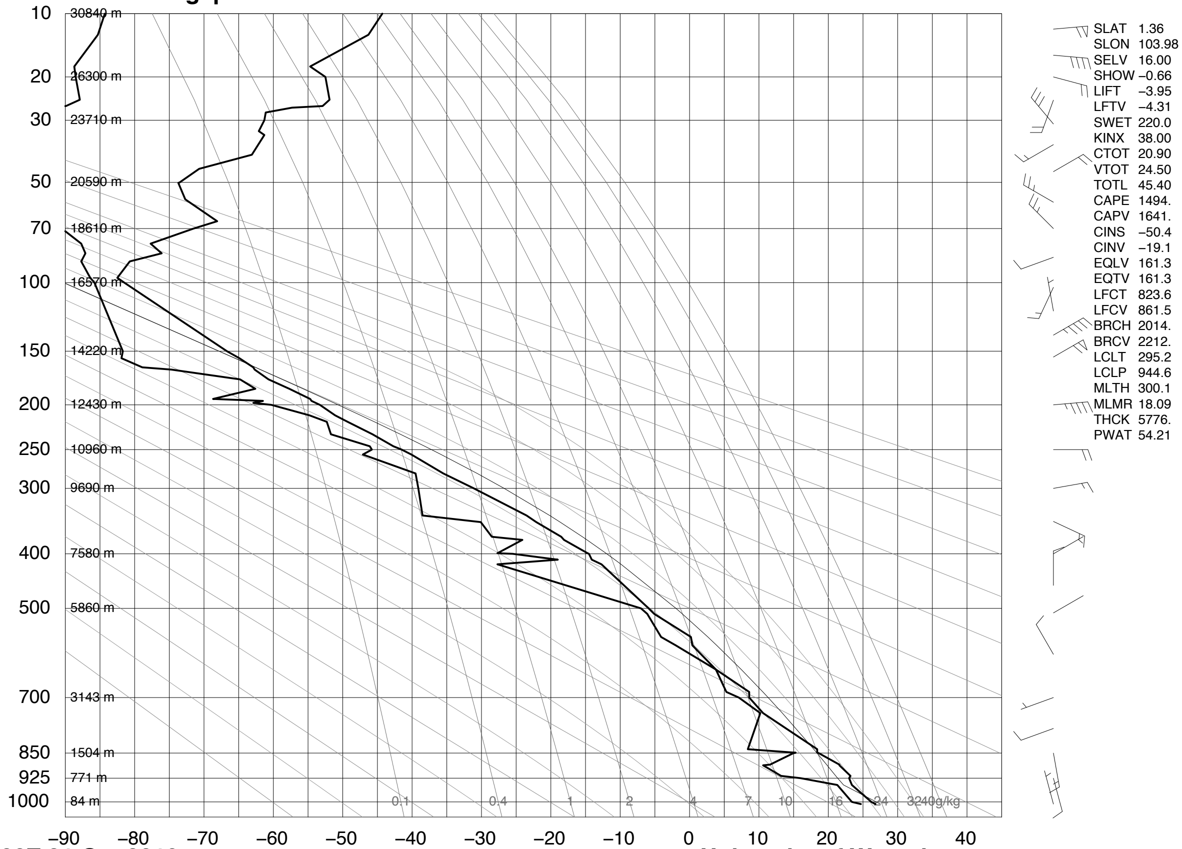
6. (1 pt) *Eddy diffusivity, mixing length theory, and the flux-gradient relationship*

It is a challenge to "close" the equation for acceleration of the mean flow, $\frac{\partial \bar{u}}{\partial t} = -\frac{\partial}{\partial z}\overline{u'w'}$, when the eddy values u' and w' are not known. Assuming that turbulent motions are oriented randomly one may use eddy mixing length theory to estimate u'. Using the definition $w' = \delta z/\delta t$, show how to express the vertical momentum flux convergence in terms of an eddy diffusivity K_{zz} and curvature of the flow: $\frac{\partial \bar{u}}{\partial t} = K_{zz}\, \frac{\partial^2 \bar{u}}{\partial z^2}$.

7. (1 pt) *Differentiating phase*

Defining the perturbation streamfunction to be $\psi'(x,t) = \psi_o e^{i(kx-\omega t)}$, show that the linearized expression for the time rate of change of vorticity following the motion can be written as $\frac{\partial \zeta}{\partial t} + \bar{u}\, \frac{\partial \zeta}{\partial x} = i\, k^2(\omega - \bar{u}\, k)\, \psi'$.

48698 WSSS Singapore



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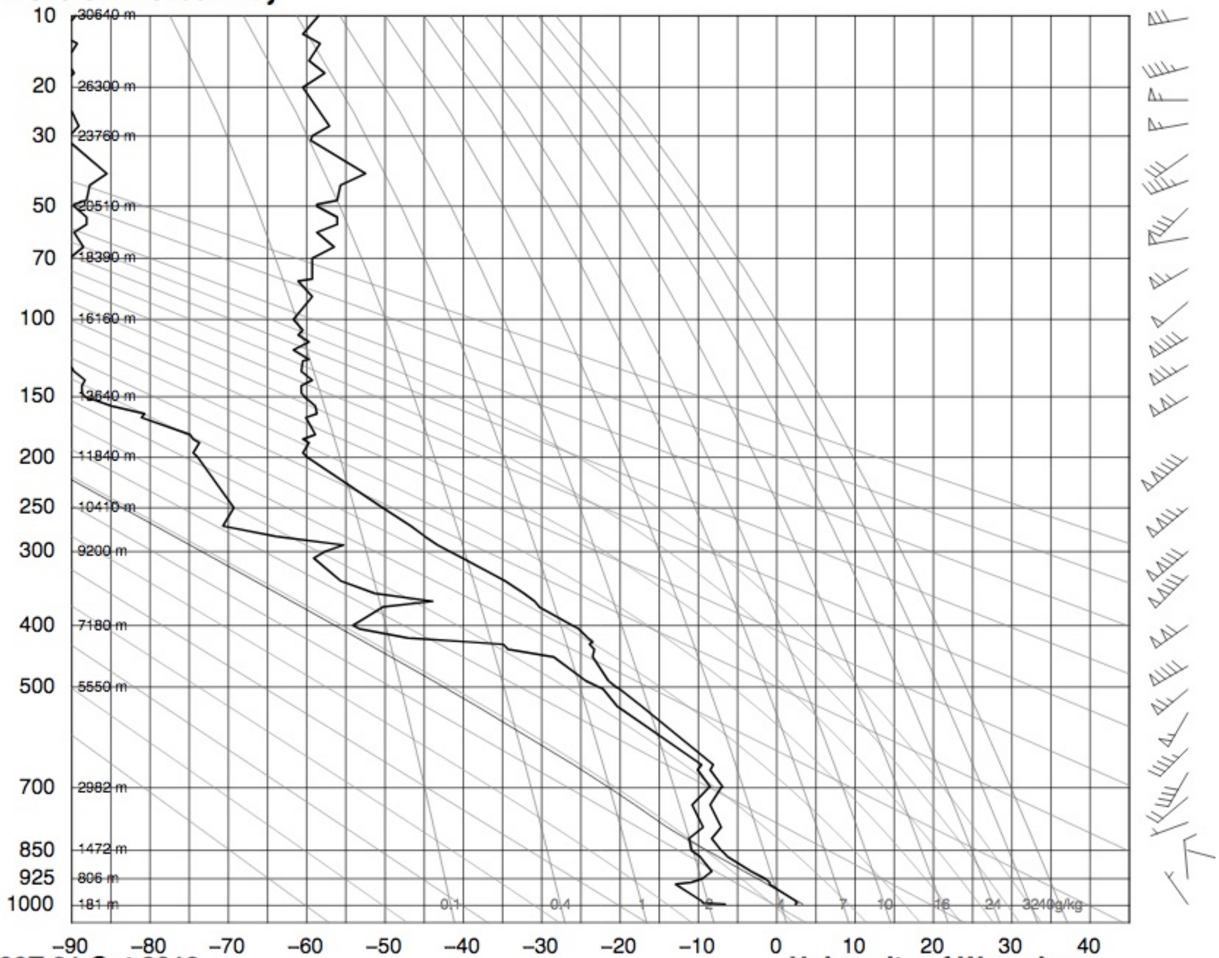

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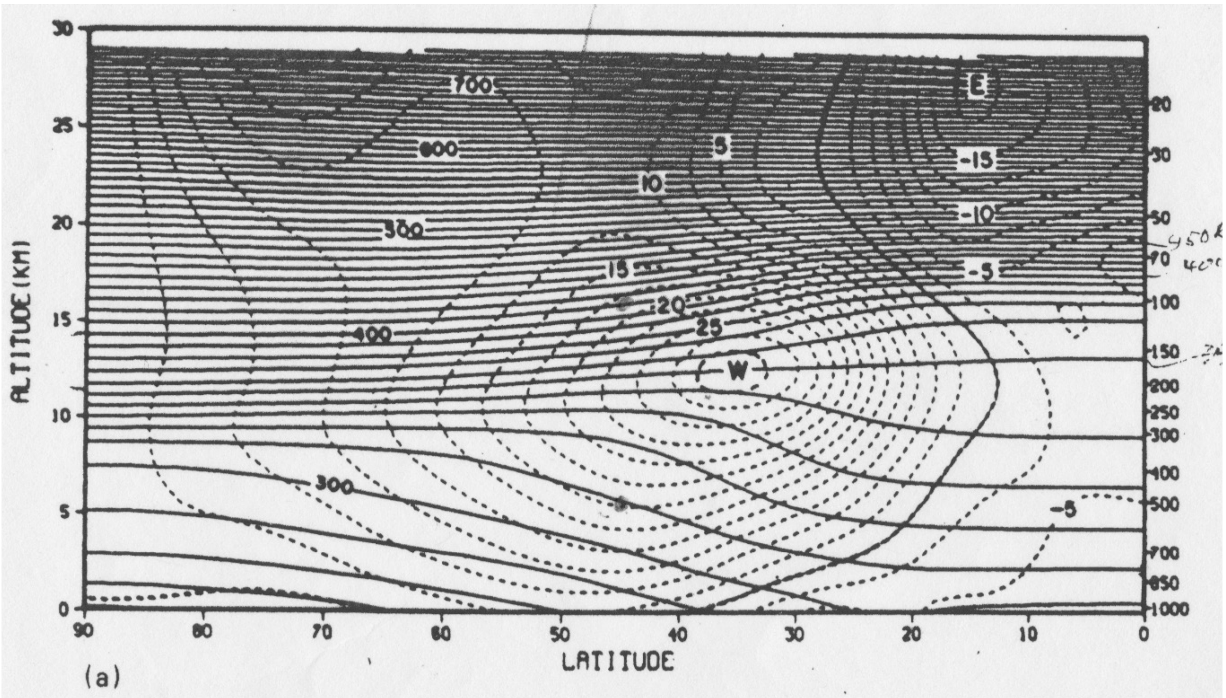
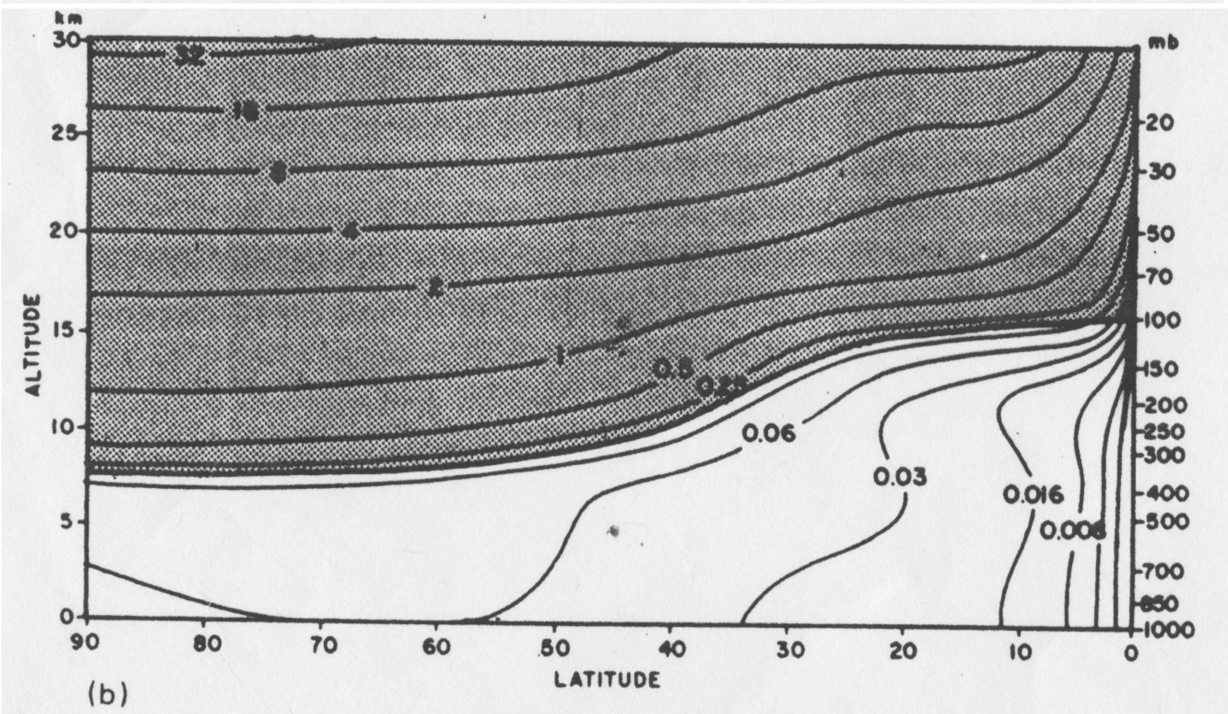

ALTITUDE (KM)
LATITUDE
700
600
500
400
300
E
W
5
10
15
20
25
-15
-10
-5
(a)


km
mb
ALTITUDE
LATITUDE
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16
8
4
2
1
0.5
0.25
0.06
0.03
0.016
0.008
(b)

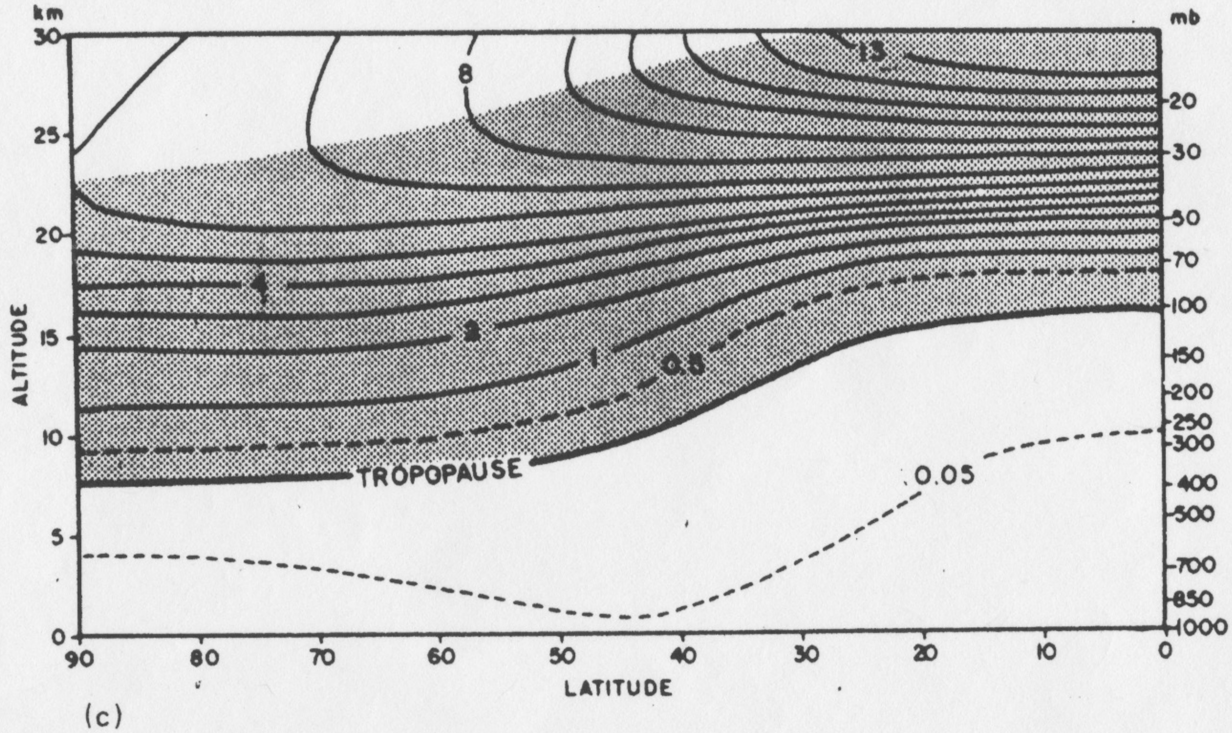

Fig. 9.1. Northern-Hemisphere zonal-annual mean cross sections for some quasi-conservative tracers. (a) Potential temperature (solid contours, kelvins) and zonal wind component (dashed contours, $m\,s^{-1}$). (b) Ertel potential vorticity in units of $10^{-5}\,K\,m^2\,kg^{-1}\,s^{-1}$. Area above the mean tropopause is shaded. (c) Ozone mixing ratio in parts per million by mass (ppmm). Shading extends from the mean tropopause to the level of maximum mixing ratio. Note that the tropopause [marked by heavy lines in panels (b) and (c)] intersects several of the potential temperature surfaces shown in panel (a). [From Danielsen (1985), with permission.]