AOS 610 GFD I Fall 2024 Prof. Hitchman

Problem Set #1 10 points
Due Thursday, September 26, 2024 (3 weeks)
Please show your work and staple pages together.

1. (1 pt) In *channel flow*, a balance between the net viscous force and pressure gradient force gives $0 = \mu \frac{\partial^2 u}{\partial y^2} + \frac{P_1 - P_2}{l}$. Assuming the no-slip condition, $u(\pm a) = 0$, and maximum flow at the center, $\frac{\partial u}{\partial y}(0) = 0$, derive a formula for the mass flow per unit depth $\int_{-a}^{a} \rho\, u\, dy$ in terms of $\frac{P_1 - P_2}{l}$, l, μ, and ρ, and half-channel width a.

2. (2 pts) *Reynolds number similarity for flow around a wire*
(Adapted from problem 18, p. 473 in Tritton.) You have taken the following measurements of drag (measured by tension on the wire) at a range of flow speeds for flow around a wire fixed at both ends, for a fluid with kinematic viscosity υ = $1.0\text{x}10^{-5}$ m^2 s^{-1}. At each of these flow speeds you find that the wire vibrates and emit sound at the Karman vortex street frequency.

U ($m\ s^{-1}$)	0.2	0.4	0.6	0.8	1.0	1.2	1.4
D (N)	0.096	0.43	1.03	1.92	3.00	4.3	5.8

a) If you replaced the fluid by one with υ = $5.0\text{x}10^{-5}$ m^2 s^{-1}, at which of the following speeds can you expect a von Karman vortex regime (and sound)?

U ($m\ s^{-1}$)	0.4	1.0	5.5
D (N)			

b) At speeds where you can predict a resonant regime, what are the values of drag in the new fluid?

3. (1 pt) *Buoyancy production of turbulence and estimating eddy diffusivity*
Watch the video (or experiment on a day with light wind). The ruler provides a spatial scale and the time is indicated in the film.
a) Estimate the width, δx in m, of the *convective* plume δt = 2 s after lighting.
b) Use scale analysis to estimate the turbulent diffusion coefficient K (m^2 s^{-1}) in a simple diffusion equation for smoke particle concentration, n (particles m^{-3}):
$\frac{\partial n}{\partial t} = K \frac{\partial^2 n}{\partial x^2}$. (Hint: In scale analysis you can cancel n on both sides.)

4. (3 pts) *Flow kinematics.* Consider *plane Couette flow* for a viscous fluid confined between two flat plates separated by distance b in the y-direction, where the steady state velocity in the x-direction is u = U y/b, U is the velocity of the upper plate relative to the stationary lower plate, and v = w = 0.

a) Find the rate of *linear strain*, rate of *shear strain*, and *vorticity*.

b) Defining the *streamfunction* by u = - $\frac{\partial \psi}{\partial y}$ and v = $\frac{\partial \psi}{\partial x}$,
express ψ in terms of U, b, and boundary condition ψ_o.

c) Sketch the *streamfunction*, labeling your axes.

5. (1.5 pts) *Ideal gas law and linearization*

a) Show that the linearized ideal gas law may be written $\frac{p'}{\bar{p}} \approx \frac{\rho'}{\bar{\rho}} + \frac{T'}{\bar{T}}$.

b) The Boussinesq approximation implies that $|\frac{p'}{\bar{p}}| \ll |\frac{T'}{\bar{T}}|$, so that $\frac{\rho'}{\bar{\rho}} \approx -\frac{T'}{\bar{T}}$.
Evaluate the validity of this assumption for synoptic scale variations by inspecting surface analyses at http://www.aos.wisc.edu/weather/Models and choosing typical mean and anomaly values for pressure and temperature. Make sure you convert to Kelvins for use in the ideal gas law.

6. (1.5 pt) *Tracer continuity equation*

Ozone mixing ratio, *r*, is observed to be 50 ppbv in Madison and the wind is from the northeast at 20 knots. Stations along the shore of Lake Michigan, 125 km to the east, report ozone observations of 100 ppbv. A photochemical sink of 2 x 10^{-4} ppbv s^{-1} applies throughout Wisconsin. Assuming that ozone varies linearly, and only in longitude ($\frac{\partial r}{\partial y} = 0$), use a finite difference form of the equation for conservation of mixing ratio, $\frac{dr}{dt} = S_r$, to forecast the ozone mixing ratio 6 hours from now in Madison. (1 m/s $\approx$1.94 knots)