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Robust GEFA Assessment of Climate Feedback to SST EOF Modes

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ABSTRACT

Atmospheric response to SST variability was estimated using generalized equilibrium feedback analysis (GEFA) in the SST EOF space with synthesis data from an idealized climate model. Results show that the GEFA atmospheric response to the leading SST EOF modes is much more accurate and robust than the GEFA feedback matrix in physical space. Therefore, GEFA provides a practical method for assessing atmospheric response to large-scale SST anomalies in terms of the leading EOFs.

Key words: generalized equilibrium feedback analysis (GEFA), assessment, climate, feedback, SST, EOF

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1. Introduction

As a generalization of the univariate equilibrium feedback assessment (EFA) of Frankignoul et al. (1998), Liu et al. (2008, hereafter Liu08) recently developed the multivariate generalized equilibrium feedback analysis (GEFA) to assess the nonlocal feedback response of an atmospheric field $\boldsymbol{A}$ to a SST (or other boundary) forcing field $\boldsymbol{T}$ in terms of the feedback matrix as

$$\boldsymbol{B} = \boldsymbol{C}_{\mathrm{AT}}(\tau)\boldsymbol{C}_{\mathrm{TT}}^{-1}(\tau)\ , \qquad (1)$$

where $\boldsymbol{C}_{\mathrm{AT}}(\tau)$ is the covariance matrix with $\boldsymbol{A}$ lagging $\boldsymbol{T}$ by $\tau>0$. τ is longer than the persistence time of the atmospheric internal variability. GEFA is related to multiple regression (Liu et al., 2008). The key of GEFA as distinguished from a simultaneous multiple linear regression (Yu et al., 2001) is that it can separate the response of the atmosphere to the ocean from the oceanic response to the atmosphere by making use of the lagged covariance. Statistically, however, as in general multiple regression, GEFA suffers from a significant sampling error when SST forcing becomes significantly correlated, e.g., for a high-resolution SST field in which SSTs become correlated among neighbors. To reduce the sampling error, the feedback matrix was proposed by Liu08 to be first estimated in the truncated SST EOF space as $\tilde{\boldsymbol{B}}_f$, where f is the truncation number of EOFs and is then converted back to the physical space (namely, estimate the atmospheric response to each grid SST) as $\boldsymbol{B}_f$. Although a proper EOF truncation improves the estimation, it remains challenging to recover the full feedback matrix in the physical space with high accuracy (Liu08).

The difficulty in estimating the feedback matrix in the physical space $\boldsymbol{B}_f$ can be seen in two examples in which GEFA is applied to a synthetic dataset derived from a 24-point stochastic climate model with atmospheric advection (see section 2). The two cases have different local ocean–atmosphere interaction coefficient λ, with λ=4 being a weak advection case and

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λ=1 being a strong advection case. A strong advection can make the atmosphere and SST more spatially correlated, and then a more ill-conditioned $\boldsymbol{C}_{\rm TT}$ is gained, which is unfavorable to estimate the feedback matrix. Figure 1 shows the pattern correlations (Fig. 1a) and amplitude ratios (standard deviation ratios) (Fig. 1b) between $\boldsymbol{B}_f$ and the true feedback matrix $\boldsymbol{B}_{\rm True}$ of the two cases. Notably, in both cases, when the EOF truncation increases, $\boldsymbol{B}_f$ is first improved, with increased correlation (Fig. 1a) and an amplitude ratio approaching 1 (Fig. 1b); then, after an optimal EOF truncation, $\boldsymbol{B}_f$ deteriorates with decreased correlation (Fig. 1a) and increased amplitude (Fig. 1b). Even with the optimal truncation, $\boldsymbol{B}_f$ remains poorly estimated, with the best correlation of 0.7 for the weak advection case (at f=4), and 0.5 for the strong advection case (at f=2).

The failure of GEFA to provide an accurate feedback matrix $\boldsymbol{B}_f$ is easy to understand. Further inclusion of EOFs beyond the optimal truncation increases the noise, such that the overall feedback matrix is no longer improved. This implies that the atmospheric response to large-scale features of SST forcing can still be estimated more accurately than the best $\boldsymbol{B}_f$ in physical space, which is precisely the point of this study.

This paper is based on Liu08. Here, we will demonstrate that, even for a highly correlated SST field with a poorly estimated $\boldsymbol{B}_f$, the atmospheric response to large-scale SST forcing, as represented by leading SST EOFs, can be estimated much more accurately. Namely, GEFA assessment of the observed atmospheric response to leading climate variability modes, such as those of Wen et al. (2010), is likely to be robust and accurate. With obvious scientific significance, the GEFA method has widespread application value and prospect (Zhu et al., 2010). Although we validated the influence of the first few SST EOF modes, the leading modes are often used to represent major climate variability modes (Shin et al., 2010), such as ENSO (El Nino and Southern Oscillation; Jiang et al., 2001), PDO (Pacific Decadal Oscillation; Zhang et al., 1997; Yang et al., 2004), AMO (Atlantic Multidecadal Oscillation; Delworth and Mann, 2000), and other research on ocean–atmosphere interaction (Wu et al., 2005). We can used linear combination to estimate the atmospheric response to a more complex SST pattern.

This study demonstrates the much-improved GEFA estimation for the leading EOF modes with synthetic data from an idealized model. The method is first briefly reviewed in section 2, and then the robust feedback estimate to EOF modes is demonstrated in section 3. A discussion and summary are given in section 4.

2. GEFA in the EOF space

In general, the quasi-equilibrium linear response of an atmospheric variability field $\boldsymbol{A}$ to a SST field $\boldsymbol{T}$ in the physical space can be written as

$$\boldsymbol{A}(t) = \boldsymbol{B}\boldsymbol{T}(t) + \boldsymbol{N}(t)\ , \tag{2}$$

where $\boldsymbol{A}$, $\boldsymbol{T}$, and $\boldsymbol{N}$ are column vectors of I dimension ($I \times 1$) for the surface air temperature, SST and stochastic atmospheric variability, respectively, and $\boldsymbol{B}(I \times I)$ is the feedback matrix. For simplicity, we have assumed the same number of points in the atmosphere and ocean. The temporal evolution of the "observed" SST field can be decomposed in its EOFs

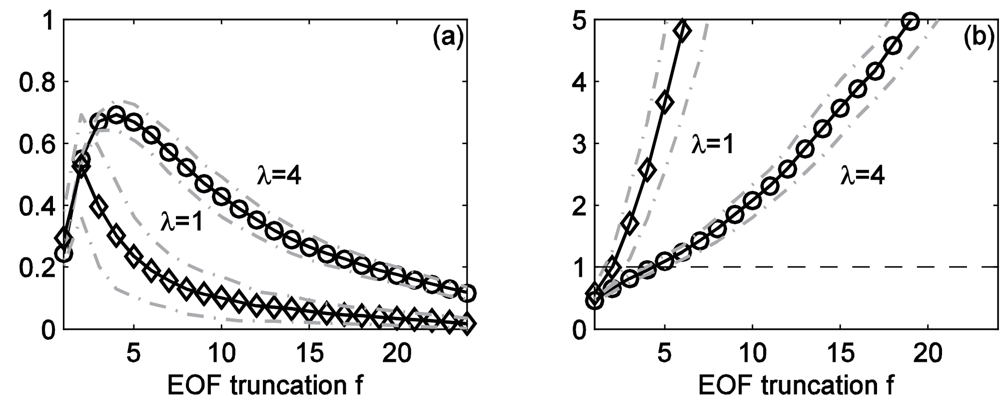

Fig. 1. The estimated feedback matrix recovered in the physical space with different SST EOF truncations (f=1 to 24) in a 24-point model for the cases of weak (λ=4, circle) and strong (λ=1, diamond) advection. The similarity with the true $\boldsymbol{B}_{\rm True}$ is shown as (a) the pattern correlation cor$\langle \boldsymbol{B}_f,\ \boldsymbol{B}_{\rm True}\rangle$, and (b) the amplitude ratio $\sigma(\boldsymbol{B}_f)/\sigma(\boldsymbol{B}_{\rm True})$. The dash lines are the 1σ ensemble spread of the 50-member ensemble.

as:

$$\boldsymbol{T}(t)=\sum_{i=1}^{I}\boldsymbol{e}_i p_i(t) \quad t=1,2,3,\cdots\ , \tag{3}$$

where $\boldsymbol{e}_i(I\times 1)$ and p_i are the ith EOF and PC, respectively. The atmospheric response in Eq. (2) can be written in the EOF space as (Wen et al., 2010):

$$\boldsymbol{A}(t)=\boldsymbol{B}\sum_{i=1}^{I}\boldsymbol{e}_i p_i(t)\boldsymbol{N}(t)\equiv\tilde{\boldsymbol{B}}\boldsymbol{P}(t)+\boldsymbol{N}(t)\ . \tag{4}$$

Here, $\boldsymbol{P}(t)=[p_1(t),p_2(t),\ldots,p_I(t)]^{\mathrm{T}}$ with the superscript T for the transpose. The feedback matrix in the EOF space is

$$\tilde{\boldsymbol{B}}\equiv[\boldsymbol{B}\boldsymbol{e}_1,\boldsymbol{B}\boldsymbol{e}_2,\cdots,\boldsymbol{B}\boldsymbol{e}_I]\ , \tag{5}$$

with the ith column $\boldsymbol{B}\boldsymbol{e}_i$ representing the atmospheric response to the ith SST EOF.

We will use the idealized stochastic coupled ocean–atmosphere model (Frankignoul and Hasselmann, 1977) as Hasselmann (1976) showed, but with an atmospheric advection, which Liu08 used. The coupled system can be written in dimensionless variables as

$$\frac{\partial T_{\mathrm{a}}}{\partial x}=\lambda(T_{\mathrm{o}}-T_{\mathrm{a}})+n(x,t)\ , \tag{6a}$$

$$\frac{\partial T_{\mathrm{o}}}{\partial t}=T_{\mathrm{a}}-T_{\mathrm{o}}\ , \tag{6b}$$

where T_{a} and T_{o} indicate the atmosphere and ocean respectively and they are of scalar type. T_{a}–T_{o} is the downward heat flux, and n is slightly different form N (see Liu08). Here λ measures the relative importance of local coupling versus nonlocal advection. A larger λ represents a stronger local coupling or a weaker advection. The sign of λ denotes the direction of advection and positive λ represents that the direction of increasing x is downwind. In the idealized model, we have a physically true response matrix; therefore, it is possible to make a comparison between the true response values and the values estimated by GEFA. However, in a realistic climate system, it is impossible to know the truth. For that reason, the robust GEFA response can only be verified in an idealized case instead of using a GCM or observation data. In a realistic application, Wen et al. (2010) shed light on the practical application of GEFA to the study of ocean–atmosphere interaction. The true surface air temperature response to an SST field can be represented by a true feedback matrix $\boldsymbol{B}_{\mathrm{True}}$, which can be derived analytically as a lower triangle matrix (see Liu08 for details). We present results for a 24-point ($I=24$) model. Many different degrees of freedom (6, 8, 12, 48, etc.) were tried, and the conclusions were essentially the same (figure omitted), so we arbitrarily chose the 24-point case as our model in this paper. The GEFA estimation was not sensitive to the atmospheric internal variability [$\sigma(n_i)$,i=1,2,. . . ,24] (figure not shown); in all the experiments of this paper, we set $\sigma(n_i)$=10 (the same as Liu08). A 50-member Monte Carlo ensemble experiment was performed to check the robustness of the GEFA response. For each member experiment, we perform a 40-year integration with the data binned into 480 months (bring the sample size close to that of practical data analysis). The monthly SST $\boldsymbol{T}$ was then decomposed into EOFs (the atmosphere was still in physical space). The true atmospheric response to the ith SST mode in this experiment can be derived by replacing $\boldsymbol{B}$ with $\boldsymbol{B}_{\mathrm{True}}$ in Eq. (5):

$$\tilde{\boldsymbol{b}}_i=\boldsymbol{B}_{\mathrm{True}}\boldsymbol{e}_i\ . \tag{7}$$

In the meantime, for each EOF truncation f ($f=1,2,\ldots,24$), the response in the truncated EOF space can be written similar to Eq. (4):

$$\boldsymbol{A}(t)=\hat{\boldsymbol{B}}_f\boldsymbol{P}_f(t)+\boldsymbol{N}(t)\ , \tag{8}$$

where $\boldsymbol{P}_f(t)=[p_1(t),p_2(t),\ldots,p_f(t)]^T$. The feedback matrix in the truncated EOF space is $\hat{\boldsymbol{B}}_f$, which can be estimated from Eq. (8) using GEFA:

$$\hat{\boldsymbol{B}}_f\equiv[\hat{\boldsymbol{b}}_1,\hat{\boldsymbol{b}}_2,\ldots,\hat{\boldsymbol{b}}_f]=\boldsymbol{C}_{AP_f}(\tau)\boldsymbol{C}^{-1}_{\mathbf{P}_f\mathbf{P}_f}(\tau)\ . \tag{9}$$

It should be noted that for different EOF truncations, the GEFA atmospheric response to a given SST mode, say the ith mode $\hat{\boldsymbol{b}}_i$, would be different. Nevertheless, the true atmospheric response to this SST mode $\tilde{\boldsymbol{b}}_i$ is unique, as in Eq. (7). Therefore, the estimated response can be compared with the truth to assess the accuracy and robustness of the GEFA response to the EOF mode.

3. Results

We first show an example of atmospheric GEFA response to leading SST EOFs and compare the GEFA response vector $\hat{\boldsymbol{b}}_i$ with the true response vector $\tilde{\boldsymbol{b}}_i$. Figure 2 shows the responses to the first (Fig. 2a), second (Fig. 2b), and third (Fig. 2c) SST EOFs for the weak advection case. The results of strong advection case are the same as those of weak one in essence, but differ in degree, so we only show the weak advection case in Fig. 2 and Fig. 3. The first three modes explain 68.4%, 15.4%, and 5.8% of the SST variance, respectively. For each mode, the GEFA responses are shown for EOF truncations of f=3, 4, and 5. Overall, all the GEFA atmospheric response vectors (marks) are in good agreement with the truth (solid

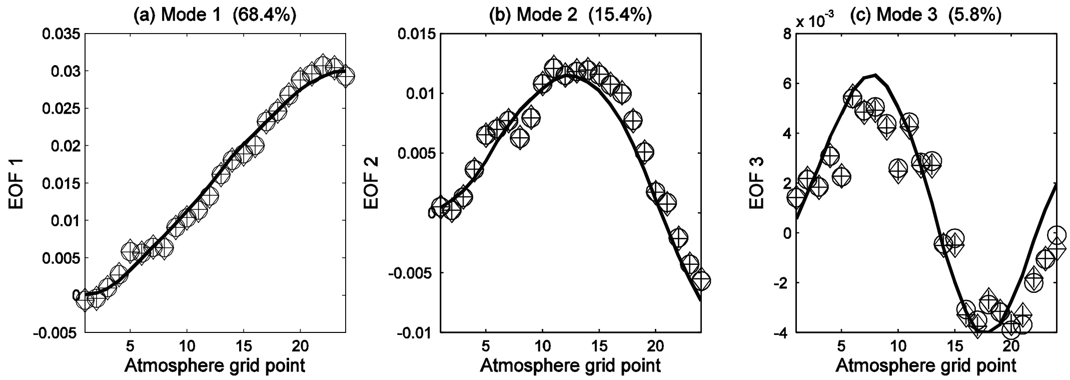

Fig. 2. In the weak advection case, a comparison between the true feedback vector (solid line) and the estimated feedback vector to SST (a) EOF 1, (b) EOF 2, and (c) EOF 3, at EOF truncations of f= 3 (circle), 4 (diamond) and 5 (plus). That is, $\tilde{\boldsymbol{b}}_1$, $\tilde{\boldsymbol{b}}_2$, and $\tilde{\boldsymbol{b}}_3$ in Eq. (7) (solid lines in each panel) are compared with the counterpart $\hat{\boldsymbol{b}}_1$, $\hat{\boldsymbol{b}}_2$, and $\hat{\boldsymbol{b}}_3$ in Eq. (9). The estimated feedback vector is insensitive to the EOF truncation, as the three marks tend to overlap with each other.

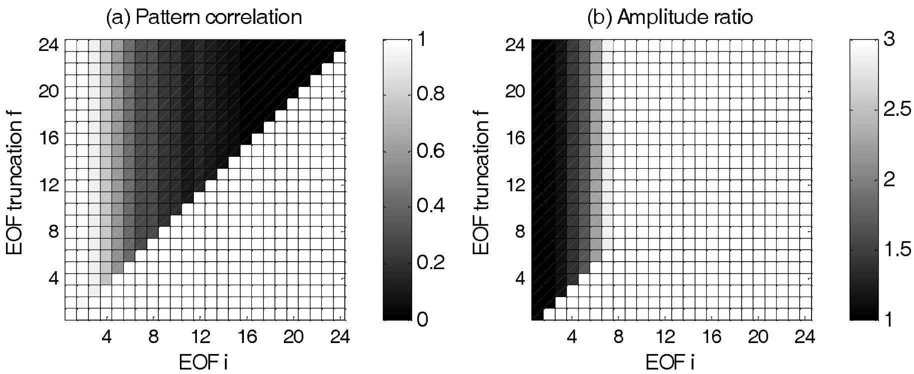

Fig. 3. Ensemble mean (of 50 realizations) (a) pattern correlation $\langle \hat{\boldsymbol{b}}_i, \tilde{\boldsymbol{b}}_i \rangle$ and (b) amplitude ratio $\sigma(\hat{\boldsymbol{b}}_i)/\sigma(\tilde{\boldsymbol{b}}_i)$ of the estimated feedback vector with the truth feedback vector for the atmospheric response to each EOF mode i, at different EOF truncation f, for the case of weak advection λ=4. (No values plotted below the diagonal line)

line), regardless of the EOF truncation. The agreement decreases slightly towards higher modes, with the pattern correlation with the truth decreasing from ∼0.99 for mode 1 to ∼0.98 for mode 2 and ∼0.96 for mode 3. However, as seen in Fig. 1, recovered in the physical space, even with the optimal truncation, the correlation of the feedback matrix with the truth is <0.7. This example demonstrates the main point of this study: in spite of the limitation of the GEFA feedback matrix in the physical space, the GEFA response to the leading EOF modes can be highly accurate. This may be because the first few SST modes capture the main SST variation characteristics that far exceed the sampling errors, while the high-order modes are minor variations, and the estimator may be intensely influenced by the sampling errors.

The favorable GEFA response to leading EOF modes is further confirmed in Fig. 3, which shows the pattern correlation (Fig. 3a) and the amplitude ratio (Fig. 3b) of the GEFA response to each SST EOF mode averaged across the 50 members for the weak advection case. In each panel, the value at row f (upward) and column i ($i \leqslant f$) is for the GEFA response to the ith mode under the EOF truncation f. Figure 3 confirms the impression from Fig. 2. First, the similarity of the GEFA response to the truth is largely independent of the EOF truncation. This is consistent with the GEFA application to the observation data shown by Wen et al. (2010). Second, the GEFA responses to the leading EOF modes are highly accurate, but they deteriorate for higher modes. For the leading 3–4 modes, the pattern correlations are all >0.9, and the amplitude ratios are <10% (<1.1) in error, much better than the best feedback matrix in the physical space (Fig. 1). However, for modes >10, the estimated response becomes poor, with the correlation <0.5 and the amplitude ratio of >100% in error (>2).

The robustness of the GEFA estimation is further

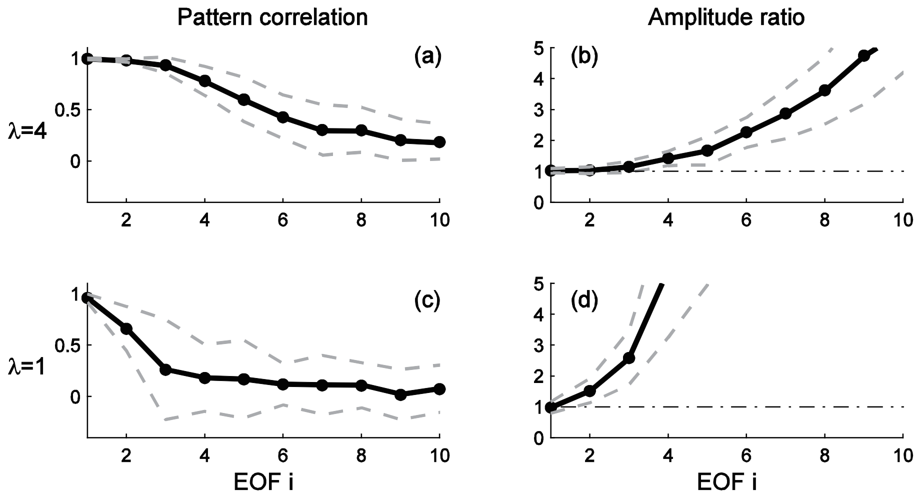

Fig. 4. The (a) pattern correlation $\langle \hat{\boldsymbol{b}}_i, \tilde{\boldsymbol{b}}_i \rangle$ and (b) amplitude ratio $\sigma(\hat{\boldsymbol{b}}_i)/\sigma(\tilde{\boldsymbol{b}}_i)$ of the estimated feedback vector with the truth feedback vector to each EOF mode i, averaged for all the EOF truncations $f(\geqslant i)$, for the case of weak advection λ=4. (c) and (d) the same as (a) and (b), respectively, but for the case of strong advection λ=1. The heavy solid and light dash lines are the ensemble mean and spread (1 σ) of the 50 realizations.

shown in Fig. 4 for both weak and strong advection cases. The 1-σ ensemble spread across the 50 members was also plotted to assess the robustness. Since the GEFA response to an individual EOF mode was largely independent of the EOF truncation (Fig. 3), the pattern correlation and amplitude ratio were averaged for all the EOF truncations (column average in Fig. 3). As discussed in Fig. 3, for the weak advection case (Figs. 4a, b), the GEFA responses to the first three EOF modes were highly accurate, with pattern correlations >0.9 and the amplitude ratio <10% in error. Furthermore, the GEFA responses were also robust, with the ensemble spread <0.1. This result was much more accurate and robust than the best feedback matrix estimated in the physical space (Fig. 1). For the strong advection (Figs. 4c, d), the response to EOF1 remained highly accurate as in the weak advection case, with a pattern correlation >0.95, amplitude ratio <5% in error, and a small ensemble spread. However, the response deteriorated rapidly toward higher EOFs. For EOF2, the pattern correlation decreased to 0.7 with a spread of ±0.2; the amplitude ratio increased to 1.4 with spread of ±0.35. Nevertheless, this was better than the best feedback matrix in the physical space (Fig. 1). For EOF3, the GEFA response was practically useless, with a pattern correlation statistically not different from zero. Thus, the GEFA response to EOF1 seems consistently highly accurate and robust. However, the GEFA responses to the next few EOF modes could be less robust if the nonlocal effect is strong and therefore should be treated with caution. Usually, however, the leading EOFs tend to be much more accurate and robust than the feedback matrix in the physical space, even if for the strong advection case.

4. Summary

Using synthetic data, we have shown that the GEFA atmospheric response to the leading SST modes, especially EOF1, is highly accurate and robust, much more so than the feedback matrix recovered in the physical space. Furthermore, the GEFA response to an SST mode is insensitive to the EOF truncations, as shown in the application to the observation data (Wen et al., 2010). The insensitivity of GEFA response to the leading SST EOF modes is a useful feature in practice, in contrast to the estimation in the physical space. There, the optimal EOF truncation for the feedback matrix in the physical space is difficult to know prior (Liu08). Therefore, a practical GEFA assessment in the physical space may be much worse than at the optimal truncation. Physically, our results imply that GEFA response to large-scale SST variability is accurate and robust, regardless of the resolution. Therefore, if we are only interested in the atmospheric response to large-scale SST variability, it is no longer necessary to find the optimal EOF truncation to achieve the optimal feedback matrix in the physical space (as Liu08). Instead, we can simply obtain the GEFA response as the sum of those on the leading EOF modes, which have significant projections on the large-scale SST pattern. Furthermore, since the leading SST EOF modes are often used to represent major climate variability modes, our study suggests that the GEFA responses to these leading climate modes, as recently studied by Wen et al. (2010), are accurate and robust.

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