# How Long is the Memory of Tropical Ocean Dynamics?*

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ABSTRACT

Combining classic equatorial wave dynamics and midlatitude wave dynamics, a unified view is proposed to account for the memory of tropical ocean dynamics in terms of oceanic basin modes. It is shown that the dynamic memory of the ocean is bounded by the cross-basin time of planetary waves along the poleward basin boundary. For realistic ocean basins, this memory originates from the mid- and high-latitude processes and is usually at interdecadal timescales.

## 1. Introduction

A fundamental issue for our understanding of interdecadal climate variability is the dynamic memory of the ocean. This issue is particularly unclear for the tropical ocean. Earlier studies of tropical oceans have focused on the upper-ocean dynamic memory that is associated with the equatorial Kelvin and Rossby waves. This tropical memory is less than interannual timescales even in the large Pacific Ocean (e.g., Cane and Moore 1981) and is unlikely to contribute significantly to interdecadal variability. A recent reexamination of the classic equatorial wave dynamics (Jin 2001, hereafter JIN) highlights the role of a class of weakly damped low-frequency tropical basin modes, the so-called scattering modes (hereafter S mode), in interdecadal tropical climate variability. These S modes appear to form a continuous spectrum with periods up to infinity. Although these S modes have been noticed long ago (Moore 1968), their physical nature remains unexplained even today. Are the S modes truly of tropical origin? Why are the S modes weakly damped with infinitely long periods? These questions are fundamental not only for the S mode itself, but also for the understanding of long-term climate variability. If the S mode is indeed of tropical origin and have periods infinitely long, the tropical ocean itself might be able to provide memory for climate variability on interdecadal and longer timescales.

This paper clarifies the nature of these S modes. The classic equatorial and midlatitude wave dynamics are reexamined from a unified perspective. We show that the S mode is identical to the planetary wave basin mode (hereafter P mode) that has been studied recently in terms of classic midlatitude wave dynamics by Cessi and Louazel (2001, hereafter CL). Therefore, in a tropical–extratropical basin, the longest dynamic memory of the ocean is associated with the P mode, whose period is determined by the transient time of planetary waves in the mid- and high latitudes.

## 2. Planetary wave basin modes

The physical nature of the P mode is well understood (see CL) and therefore can be used later to understand the S mode. With the long-wave approximation, the linearized shallow-water system on an equatorial $\beta$ plane can be written as (JIN)

$$\partial_t u - yv = -\partial_x h, \qquad yu = -\partial_y h,$$
$$\partial_t h + \partial_x u + \partial_y v = 0. \tag{1}$$

This system has been nondimensionalized by the equatorial deformation radius $L_D = (c_0/\beta)^{1/2}$ for $y$, the basin width $L$ for $x$, the Kelvin wave crossing time $L/c_0$ for $t$, the mean depth $D$ for $h$, the Kelvin wave speed $c_0$ and $c_0 L_D/L$ for $u$ and $v$ ($c_0 = \sqrt{g'D}$ is the gravity wave speed). We will consider an ocean basin bounded by $x = 0$ and 1 and $y = Y_s$ and $Y_n$.

Replacing the $u$ equation in (1) by geostrophy, we

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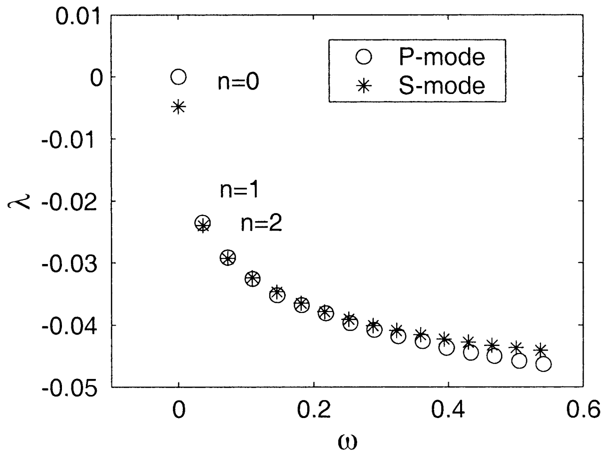


FIG. 1. The eigenvalues of the first 16 P (circles) and S modes (asterisks) for a basin of $Y_N = 13.23$. The P and S modes almost coincide with each other, especially for the lower modes. (S mode 0 corresponds to the trivial P mode 0 and has a damping rate much smaller than other S modes.) The truncation number for the S mode is $N = 44$ according to (6). With typical parameters of $\beta = 2 \times 10^{-11}$ m$^{-1}$ s$^{-1}$, $D = 400$ m, and $g' = 0.02$ m s$^{-2}$, $L_D$ is about 377 km and therefore the dimensional $Y_N \sim 5000$ km. For the Pacific basin of $L \sim 10\,000$ km, the dimensional time unit $L/c_0$ is about 2 months. It should also be pointed out that the eigenvalues are determined mainly by planetary waves on $Y_N$. Further calculations of the P mode find no substantial changes of the eigenvalues if $Y_S$ is moved north of the equator (not shown).

have the classic planetary wave equation in the extratropics $\partial_t h - (1/y^2)\partial_x h = 0$, where $1/y^2$ is the westward planetary wave speed along latitude $y$. Assuming the form of $h = \exp(\sigma_P t)H(x, y)$, the eigenfunction $H$ and eigenvalue $\sigma_P = \lambda_P + i\omega_P$ are determined by the eastern boundary condition $u|_{x=1} = 0$ as

$$H = \exp[\sigma_P(x - 1)y^2], \tag{2a}$$

and furthermore by the basinwide mass conservation $\iint h\, dx\, dy = 0$ as

$$\int_{Y_S}^{Y_n} [1 - \exp(-\sigma_P y^2)]/y^2\, dy = 0. \tag{2b}$$

The eigenvalue problem (2) formally applies to a basin that includes the equator, because, toward the equator, the integrand in (2) as well as all the eigenfunctions remain finite. Below, we will set $Y_S = 0$ on the equator to focus on the hemispherically symmetric modes. There are infinitely many eigenvalues $\sigma_{Pj} = \lambda_{Pj} + i\omega_{Pj}$($j = 0, 1, 2, \ldots \infty$). For $Y_N > O(1)$, the frequencies can be approximated as $\omega_{Pj} = j\omega_{P1}$, where the fundamental frequency $\omega_{P1}$ corresponds to the cross-basin time of a planetary wave along $Y_N$, or

$$\omega_{P1} = 2\pi/Y_N^2. \tag{3}$$

Calculations show that, except for the trivial solution $\sigma_{P0} = 0$, all the eigenmodes are damped ($\lambda_{Pj} < 0$), as seen in the example in Fig. 1. Furthermore, these modes are weakly damped in the sense that their damping rates tend to be smaller than their frequencies. The eigenfunction is characterized by an increasing amplitude toward the northwest corner of the basin (Figs. 2c,g). The contribution to the mass conservation comes predominantly from the large amplitude of $h$ at high latitudes. This suggests that these modes originate from, or are determined by, the planetary waves in the mid- and high latitudes. The phase line follows the westward planetary wave speed (Figs. 2d,h), tilting northeastward with latitude. This tilting results in an apparent zonal wavenumber that increases with latitude: the zonal wavenumber is quantized along $Y_N$ with little net cross-boundary transport, but always becomes zero along the equator with no cross-equator transport at any point.

The formation of these P modes can be understood from the basinwide mass adjustment in terms of planetary waves and coastal Kelvin waves (Liu et al. 1999). When a planetary wave hits the western boundary, its mass is quickly transferred to the eastern boundary by the coastal and equatorial Kelvin waves along the basin boundary and equator, respectively, and is then radiated westward by the secondary planetary waves generated along the eastern boundary. If the planetary wave speed is independent of latitude, all the eastern boundary planetary waves reach the western boundary at the same time and the entire mass redistribution process repeats itself exactly, forming a neutral basin mode. When the planetary wave speed varies with latitude, the final normal mode decays, because of the distortion of the wave front after each reflection on the western boundary. The waves are therefore damped even without explicit interior dissipation.

## 3. Equatorial scattering modes and its physical nature

In classic equatorial wave dynamics (Cane and Sarachik 1981), the solution that satisfies eastern boundary condition $u|_{x=1} = 0$ can be represented as the sum of one equatorial Kelvin wave and $N$ equatorial long Rossby waves. Assuming the form of solution of $h = \exp(\sigma_S t)H(x, y)$, we have (JIN) the eigenfunction $H = (q + p)/2$,

$$q = \sum_{n=0}^{N} q_{2n}(x)\psi_{2n}(y), \qquad p = \sum_{n=0}^{N-1} p_{2n}(x)\psi_{2n}(y), \tag{4a}$$

$q_{2n} = \sqrt{(2n - 1)!!/(2n)!!}\exp\{\sigma_S[4n(x - 1) - x]\}$ and $p_{2n} = \sqrt{(2n + 2)/(2n + 1)}q_{2n}$. The eigenvalue $\sigma_S = \lambda_S + i\omega_S$ is further determined by the absence of net-mass flux on the western boundary $\int_{-\infty}^{\infty} u|_{x=0}\, dy = 0$ as

$$1 - \sum_{n=1}^{N} \frac{(2n - 3)!!}{(2n)!!}\exp(-4n\sigma_s) = 0. \tag{4b}$$

For a finite $N$, (4b) has infinitely many damped eigenmodes. The $N$ slowest modes are the S modes of $k = 0$: $\sigma_{Sj} = \lambda_{Sj} + i\omega_{Sj}$($j = 0, \pm1, \pm2, \ldots \pm N/2$), which have no node point along the equator. These modes have

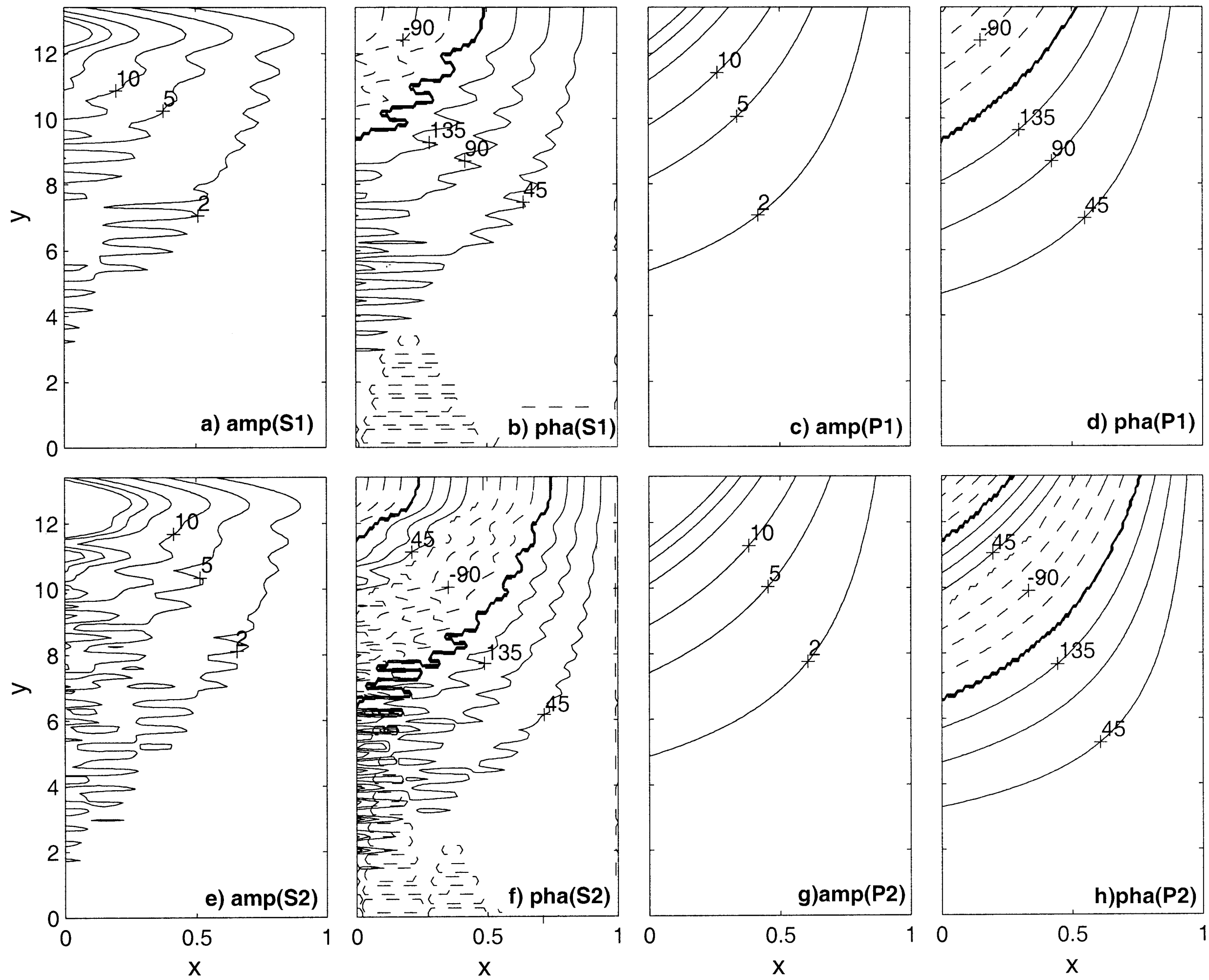


FIG. 2. Eigenfunctions of the first and second modes (eigenvalues are shown in Fig. 1): (a) amplitude and (b) phase of the first S mode; (c) amplitude and (d) phase of the first P mode; (e) amplitude and (f) phase of the second S mode; (g) amplitude and (h) phase of the second P mode. Each $H$ field has been normalized by its value as $x = 1$ and $y = 0$. Again, the large-scale structures of the P and S modes agree well. Contours are 2, 5, 10, 20, 30, and 40 for the amplitude and the contour interval is 45° for the phase. The direction of increasing phase points to the direction of wave propagation. Negative phases are in dashed lines. (The phase has a 360° discontinuity at ±180°.) The small-scale features in the S mode imply an important contribution of higher-order meridional modes.

been suggested as important for tropical interdecadal variability by JIN. Calculations show that, for $N > O(1)$, the frequencies of these modes can be approximated as $\omega_{Sj} = j\omega_{S1}$, where the fundamental frequency is

$$\omega_{S1} = \pi/(2N). \tag{5}$$

The frequencies of the S modes are bounded by half of $\omega_{SN} = \pi/2$, where $\pi/2$ corresponds to 4 times the transient time of the equatorial Kelvin wave and is the frequency of the slightly leaky equatorial basin mode of Cane and Moore (1981). When $N \to \infty$, all the $\lambda_{Sj}$ approach zero, such that these S modes form an infinite set of almost-neutral modes.

The major contribution of this paper is to demonstrate that these slow S modes are of the same nature as the P modes discussed before. The key condition for the existence of the P mode is the basinwide mass conservation. Since the equatorial wave solution (4) already satisfies no-net transport across the eastern, western, and southern (equator) boundaries, the mass conservation can be achieved by an additional constraint of no-net transport across $Y_N$. This can be accomplished by limiting $N$ such that $2N + 1$ is $O(Y_N^2)$; that is, only summing up Rossby waves with critical latitudes south of $Y_N$. [In previous works (e.g., Cane and Sarachik 1981), a similar $N$ was selected, but according to frequency, not the latitude of the northern boundary.] Since the amplitude of the Rossby waves of $n$ ($\leq N$) virtually vanish beyond $Y_N$ in the interior ocean, this finite truncation guarantees no-net-mass flux across $Y_N$. More precisely, $N$ should be (the integer part of)

$$N = Y_N^2/4, \tag{6}$$

such that the fundamental frequencies of the S and P

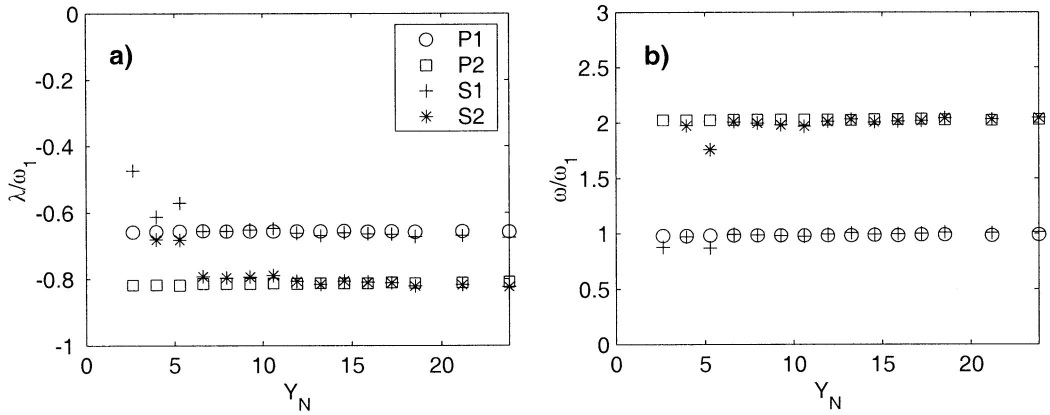


FIG. 3. Normalized eigenvalues of the first two modes for different northern boundaries: (a) damping rates and (b) frequencies for the P mode (first mode, circle; second mode, square) and S mode (first mode, plus sign; second mode, asterisk). The eigenvalues are normalized by the fundamental frequency (3) for the P mode and (5) for the S mode. The truncation numbers of the S mode are calculated following (6) as $N = [Y_N^2/4] + 1$, where [ ] is the integer part. The good agreement between the P and S modes is clearly shown. The most important feature is the implied diminishing of eigenvalues with increased $Y_N$. Similar results are obtained by direct numerical eigenvalue calculation of the linear shallow-water system in its finite difference from Yang and Liu (2002, manuscript submitted to *J. Phys. Oceanogr.*).

modes are identical: $\omega_{S1} = \omega_{P1}$ As such, the *j*th S mode would have approximately the same frequency as the *j*th P mode. Calculation further shows that the damping rates of the *j*th S and P modes are also almost identical (see the example in Fig. 1). The large-scale structures of the S mode 1 and P mode 1 are also in good agreement (Figs. 2a–d): both have an amplitude increasing toward the northwest corner, a westward propagation, and a zonal wavenumber 1 along the northern boundary. A similar agreement can be found between the second S and P modes (Figs. 2e–h). Finally, the S and P modes also show good agreement for basins of different northern boundaries. For $Y_N$ increasing from about 3 to 24, the damping rate (Fig. 3a) and frequency (Fig. 3b) of P mode 1 and 2 remain almost unchanged, after being normalized by the fundamental frequency (3). For a given P mode, therefore, its damping rate and frequency diminish as $1/Y_N^2$ when $Y_N \to \infty$. These fundamental features are also captured by the S mode for a wide range of truncation $N$ (from 3 to 143). The agreement between the S and P modes are especially good for a basin whose northern boundary extends beyond the Tropics ($Y_N > 5$).

The agreement between the S and P modes suggests that the two modes are of the same nature [as long as $Y_N > O(1)$]. Therefore, major features of the S mode can be easily understood in terms of the P mode. It is now clear that the S modes are well-defined for a basin of a finite size. The continuous spectrum of S modes as $N \to \infty$ is an artifact of the P mode on an infinite $\beta$ plane with $Y_N \to \infty$ and will never be realized in a finite basin. Most importantly, the longest period of a nontrivial S (or P) mode is the cross-basin time of the planetary wave on $Y_N$. This longest memory of the ocean basin is usually at interdecadal or longer timescales for a realistic tropical–extratropical basin whose poleward boundary is located at the mid- to high latitudes.

The agreement between the P and S modes are expected, physically, in spite of the dramatic difference in their mathematical appearances [(2) versus (4)]. Indeed, the only large-scale S-mode component that is missing in the P mode is the equatorial Kelvin wave. At very low frequencies, however, the role of the equatorial Kelvin wave is limited mainly to the eastward mass transfer along the equator, analogous to what a coastal Kelvin wave does along the southern boundary of a midlatitude basin (Liu et al. 1999). At higher frequencies [$\omega > O(\pi/2)$], the equatorial Kelvin wave becomes distinctively important. Now, there are infinitely many higher-frequency equatorial modes that are contained in the equatorial eigenvalue problem (4) (see JIN), but are absent in the P-mode equation (2). These high-frequency modes have zonal wavenumbers larger than 1 and their formation depends critically on the reflection of equatorial Kelvin and Rossby waves within the equatorial waveguide (Cane and Moore 1981).

The S mode, in a sense, is an inefficient representation of the P mode, because of the critical importance of the higher meridional modes in the representation of the P mode. For example, in the case of $N = 44$, major contributions to the eigenfunction maximum near the northern boundary come from the modes higher than 20–30 (Fig. 4). This is in contrast to those equatorial basin modes of higher frequencies that are contributed mostly by the first few meridional modes and tend to be equatorially trapped (Cane and Moore 1981).

## 4. Summary

We have focused on the nature of the ultra-low-frequency basin modes in a tropical–extratropical basin.

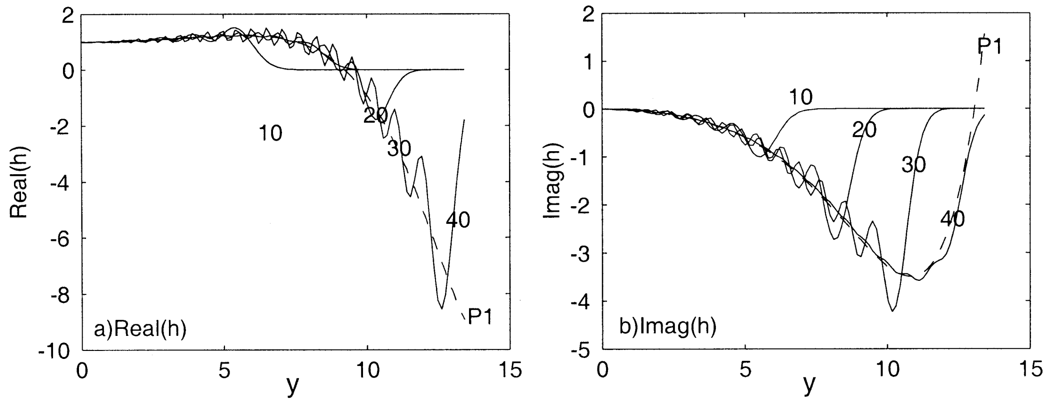


FIG. 4. The (a) real and (b) imaginary parts of the eigenfunction $h$ across the midbasin at $x = 0.5$ of the S mode according to (6) with a different number of meridional modes of summation (for the first mode in Figs. 2a,b). The eigenvalue is calculated with $N = 44$ and the summation for $h$ is to 10, 20, 30, and 44 as labeled. The P-mode solution (in Figs. 2c,d) is also plotted (dashed line) for comparison. It is seen that higher modes play the most important role for the maximum $h$ at high latitudes.

With the truncation number of the equatorial Rossby waves determined by the northern boundary as in (6), the equatorial wave dynamics is unified with the midlatitude wave dynamics in the low-frequency limit. A low-frequency basin mode can be represented either as the P mode from the perspective of midlatitude wave dynamics, or as the S mode from the perspective of the equatorial wave dynamics. The longest period of the basin modes equals the transient time of the planetary wave at the northern boundary (more precisely, at the northernmost latitude that the eastern boundary coastal Kelvin wave can reach). Therefore, the maximum memory of the tropical ocean is always finite and is determined by the extratropical Rossby waves. One can conclude that, for climate variability of timescales longer than interannual (or several transient times of the equatorial Kelvin wave), the contribution of the ocean dynamic memory can come only from outside the Tropics. This conclusion, of course, is valid in the context of linear ocean dynamics and for a fixed baroclinic mode. The tropical ocean may also gain memory locally from higher baroclinic ocean modes (e.g., Liu et al. 2002), nonlinearity (e.g., Munnich et al. 1991) and ocean–atmosphere coupling (e.g., JIN). The relative contribution of these elements to the observed interdecadal climate variability remains to be studied.

There are now two mechanisms proposed that emphasize the role of extratropical–tropical oceanic teleconnection in providing the memory for tropical decadal variability in the coupled ocean–atmosphere system: the mean advection mechanism (Gu and Philander 1997) and perturbation advection mechanism (Kleeman et al. 1999). The former is associated with higher baroclinic planetary wave modes in the presence of mean thermocline flow, while the latter, perhaps, the first baroclinic planetary wave mode (Liu 1999). The basin mode proposed here is related to the first baroclinic mode, because of its independence of the mean thermocline flow, and therefore can perhaps shed light on the mechanism of Kleeman et al. (1999) from the perspective of ocean dynamics. Nevertheless, the Gu and Philander (1997) mechanism could be related to basin modes of higher baroclinic modes in the presence of mean flow, if they do exist.

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